

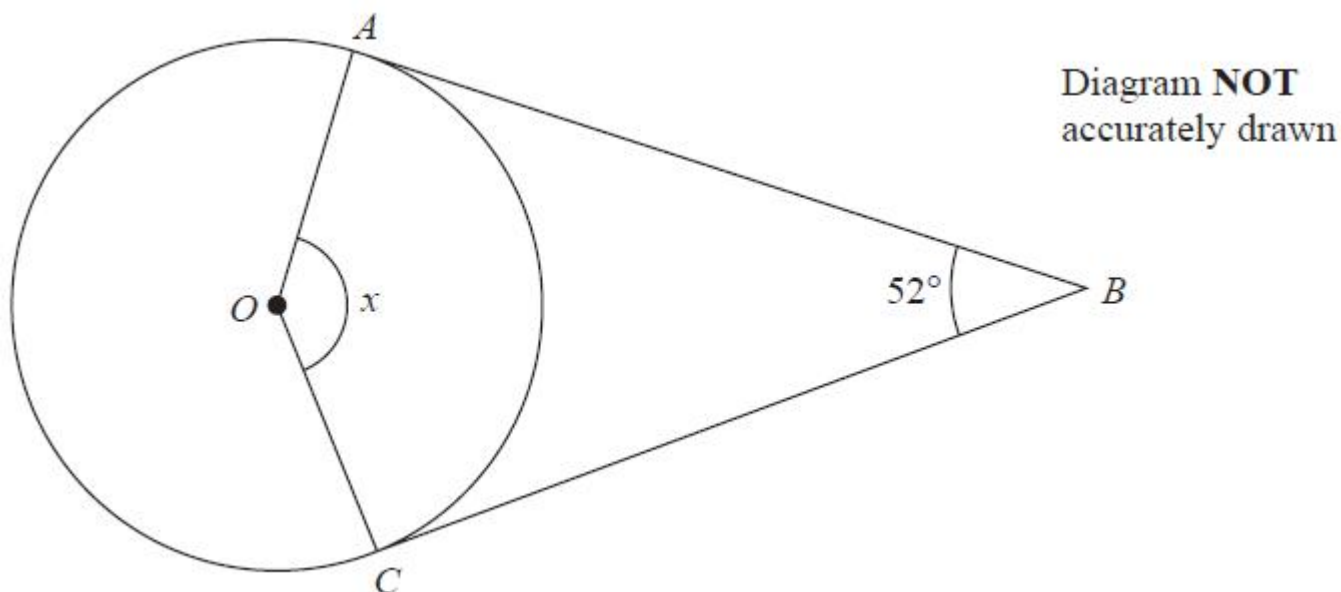


Unit 2 Revision Sheet E Circle Theorems Higher

Note: Higher tier students must also revise using the foundation tier revision worksheets as this content can also be assessed on higher tier papers.

Questions

Q1.

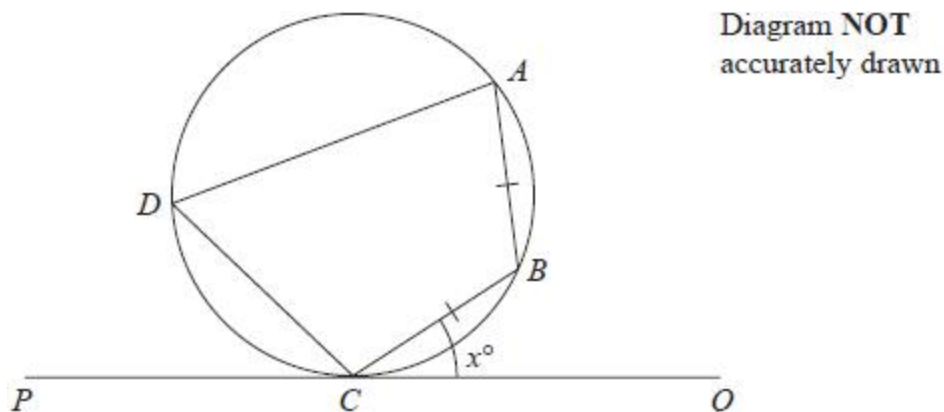


A and C are points on a circle, centre O.
AB and CB are tangents to the circle.
Angle $ABC = 52^\circ$

Work out the size of angle x .
Give a reason for each stage of your working.

(Total for question = 4 marks)

Q2.





A, B, C and D are points on a circle.
 PCQ is a tangent to the circle.
 $AB = CB$.

Angle $BCQ = x^\circ$

Prove that angle $CDA = 2x^\circ$

Give reasons for each stage in your working.

(Total for question = 5 marks)

Q3.

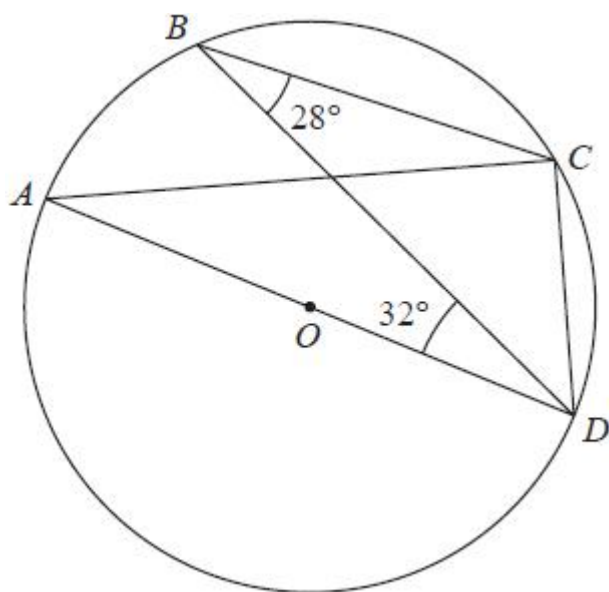


Diagram **NOT**
accurately drawn

A, B, C and D are points on a circle, centre O .
 AOD is a diameter of the circle.

Angle $CBD = 28^\circ$

Angle $BDA = 32^\circ$

Find the size of angle BDC .

Give a reason for each stage of your working.

(Total for question = 4 marks)



Q4.

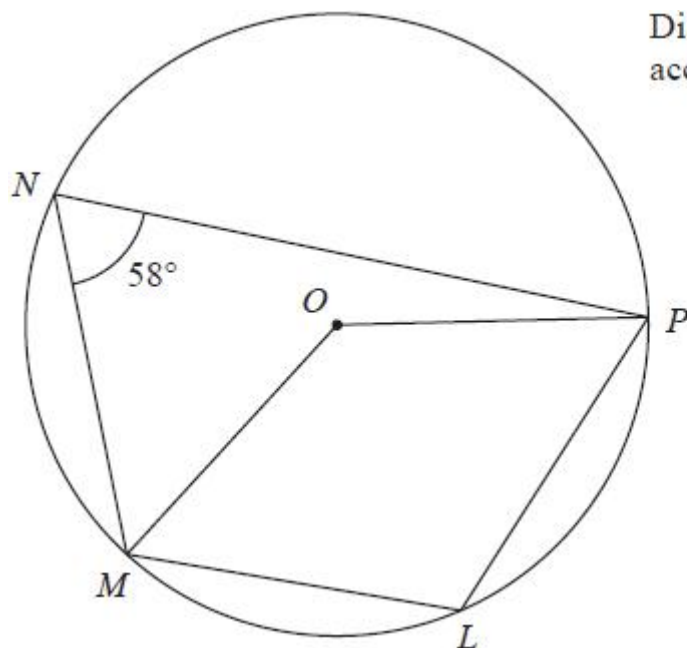


Diagram NOT
accurately drawn

L, M, N and P are points on a circle, centre O

Angle $MNP = 58^\circ$

- (a) (i) Find the size of angle MLP
(ii) Give a reason for your answer.

(2)

- (b) Find the size of the reflex angle MOP

(2)

(Total for question = 4 marks)

Q5.

P, Q, R, S and T are points on a circle with centre O .

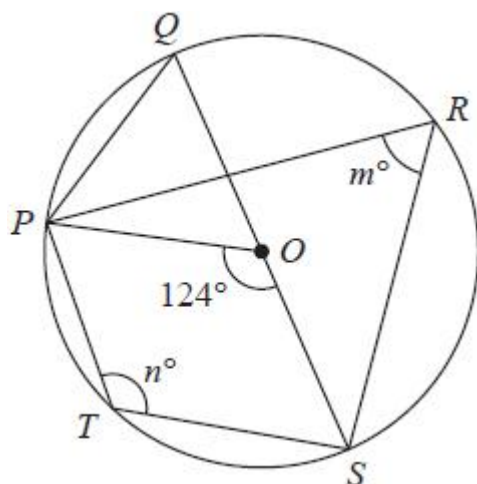


Diagram NOT
accurately drawn



QOS is a diameter of the circle.

angle $POS = 124^\circ$ angle $PRS = m^\circ$ angle $PTS = n^\circ$

(a) Find the value of

- (i) m
- (ii) n

(2)

(b) Find the size of angle QPO .

(1)

(Total for question = 3 marks)

Q6.

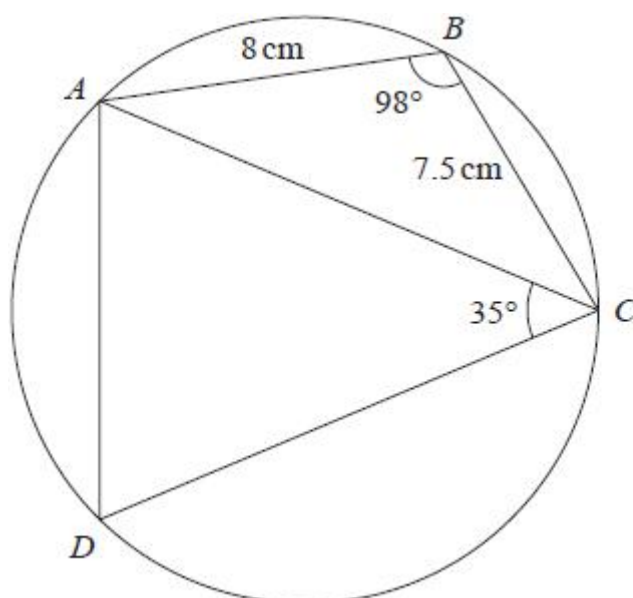


Diagram NOT
accurately drawn

$ABCD$ is a quadrilateral where A , B , C and D are points on a circle.

$AB = 8$ cm

$BC = 7.5$ cm

Angle $ABC = 98^\circ$

Angle $ACD = 35^\circ$

Work out the perimeter of quadrilateral $ABCD$.
Give your answer correct to one decimal place.

(Total for question = 6 marks)



Q7.

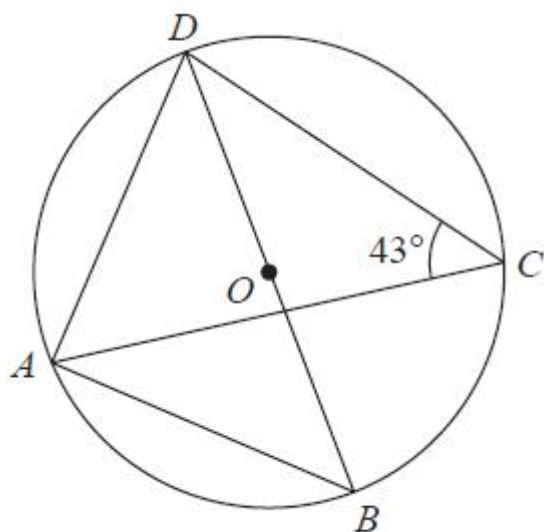


Diagram **NOT**
accurately drawn

A , B , C and D are points on a circle, centre O .
 DOB is a diameter of the circle.
Angle $ACD = 43^\circ$

Work out the size of angle ADB .
Give a reason for each stage in your working.

(Total for question = 5 marks)

Q8.

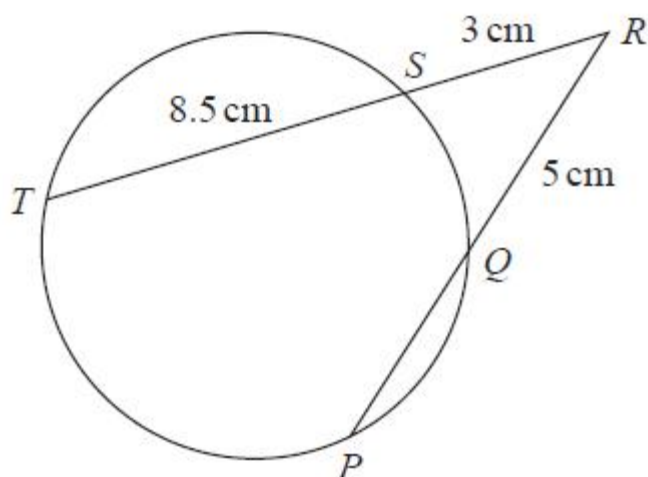


Diagram **NOT**
accurately drawn

P , Q , S and T are points on a circle.
 TSR and PQR are straight lines.

Work out the length of PQ .

(Total for question = 3 marks)



Q9.

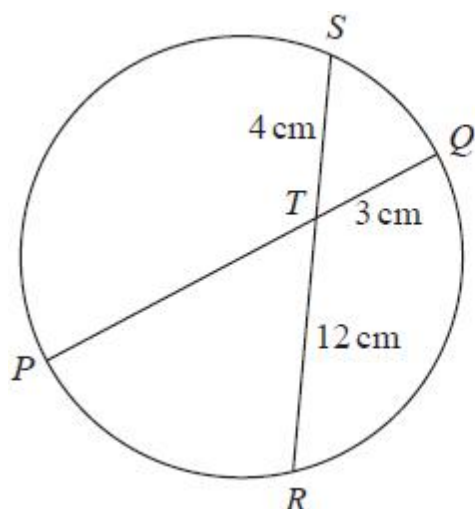


Diagram NOT
accurately drawn

PTQ is a diameter of a circle.
 RTS is a chord of the circle.

$$TQ = 3 \text{ cm} \quad ; \quad ST = 4 \text{ cm} \quad ; \quad TR = 12 \text{ cm}$$

Calculate the radius of the circle.

(Total for question = 3 marks)

Q10.

A , B and C are points on a circle, centre O

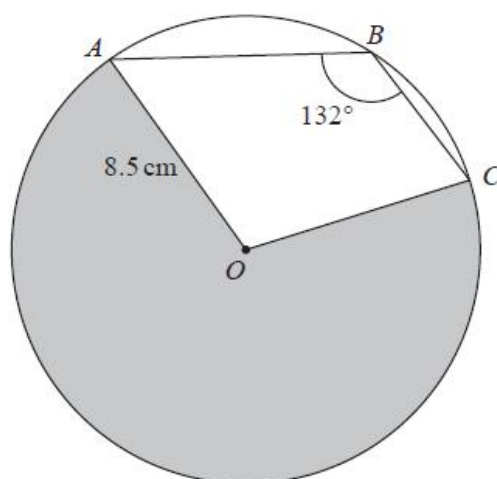


Diagram NOT
accurately drawn

The radius of the circle is 8.5 cm
Angle $ABC = 132^\circ$

Work out the perimeter of the shaded sector AOC
Give your answer correct to 3 significant figures.

(Total for question = 3 marks)



Mark Scheme

Q1.

Q		Working	Answer	Mark	Notes	
		$(x =) 360 - (90 + 90 + 52)$	128 Correct reasons	4	M1 A1 B1 B1	The angle between a tangent and a radius is 90° oe Angles in a quadrilateral add up to 360° oe
						Total 4 marks



Q2.

Question	Working	Answer	Mark	Notes
	angle $CDB = x$ or angle $CAB = x$ angle $CBA = 180 - 2x$ angle $CDA = 180 - (180 - 2x) = 2x$ Alternative method angle $CDB = x$ or angle $CAB = x$ angle $ACB = x$ angle $ACQ = 2x$ and angle $CDA = 2x$	proof with reasons	5	M1 M1 M1 B1 dep on M1 for any one appropriate circle theorem reason A1 for complete proof with full reasons <u>alternate segment theorem</u> , angles in a <u>triangle</u> sum to <u>180°</u> , <u>isosceles triangle</u> , <u>opposite</u> angles of a <u>cyclic quadrilateral</u> sum to <u>180°</u> M1 M1 M1 B1 dep on M1 for any one appropriate circle theorem reason A1 for complete proof with full reasons <u>alternate segment theorem</u> , <u>isosceles triangle</u>
		proof with reasons	5	B1 dep on M1 for any one appropriate circle theorem reason A1 for complete proof with full reasons <u>alternate segment theorem</u> , <u>isosceles triangle</u>



	<p>Alternative method angle $OCB = 90 - x$ angle $BOC = 180 - 2(90 - x) (=2x)$ angle $AOB = 2x$ and angle $CDA = 2x$</p> <p>eg angle $ABC = 180 - 2x$ Angle $CAB = \text{angle } ACB =$ $[180 - (180 - 2x)] \div 2 = x$ $BCQ = CAB = x$</p>	<p>proof with reasons</p>	<p>5</p>	<p>M1 M1 M1</p> <p>B1 dep for any one appropriate circle theorem reason A1 for complete proof with full reasons angle between <u>tangent and radius</u> is 90° oe, angles in a <u>triangle</u> sum to 180°, <u>isosceles triangle</u>, angle at <u>centre is twice</u> angle at <u>circumference</u> oe</p> <p>M1 M1</p> <p>M1 B1 Dep on M1 for any one appropriate circle theorem reason A1 For complete proof with reasons e.g. <u>opposite angles</u> of <u>cyclic quadrilateral</u> sum to 180° angles in <u>triangle</u> sum to 180° <u>isosceles triangle</u> <u>alternate segment</u> theorem</p>
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Q3.

Question	Working	Answer	Mark	Notes
	Angle $CAD = 28^\circ$ or angle $ACB = 32^\circ$ or angle $ACD = 90^\circ$ or angle $ABD = 90^\circ$		4	M1
		30°		A1 For a correct answer of 30
	<u>Angles</u> in the <u>same segment</u> are equal, <u>angle</u> in a <u>semicircle</u> is 90° (or <u>angle</u> at centre is <u>double</u> angle at <u>circumference</u> oe) angles in a <u>triangle</u> add up to <u>180°</u> / <u>angles</u> in a <u>triangle</u> <u>isosceles</u> triangle <u>alternate</u> angles vertically <u>opposite angles</u> (or <u>vertically opposite</u>) <u>angles</u> at a <u>point</u> <u>opposite angles</u> in a <u>cyclic quadrilateral</u> angle between <u>tangent</u> and <u>radius</u> (<u>diameter</u>) <u>alternate segment</u> theorem <u>angles subtended</u> by the <u>same arc</u> (or <u>chord</u>) at the <u>circumference</u> (or <u>on the circle</u>)			B2 Dep on M1 for all correct reasons for their method used (if not B2 then award B1(dep on M1) for a correct circle theorem reason)
				<i>Total 4 marks</i>



Q4.

Q	Working	Answer	Mark	Notes
(a)(i)		122	1	B1
(a)(ii)		reason	1	B1 (dep on a correct answer or a correct method seen for (i)) <u>Opposite angles</u> in a <u>cyclic quadrilateral</u> sum to 180°
(b)	$360 - 2 \times 58$ or $2 \times '122'$		2	M1 ft from (a)
		244		A1
				Total 4 marks

Q5.

Q	Working	Answer	Mark	Notes
(a) (i)		62	3	B1
(a) (ii)		118		B1ft 180 – their (a)(i)
(b)		62		B1
				Total 3 marks



Q6.

Q	Working	Answer	Mark	Notes
	[ADC =] $180 - 98 (= 82)$		6	M1 may be seen on diagram
	[AC ² =] $8^2 + 7.5^2 - 2 \times 8 \times 7.5 \times \cos(98) (= 136.95\dots)$			M1 correct equation for AC or AC ²
	[AC =] $\sqrt{136.95}$ or $\sqrt{64 + 56.25 + 16.7\dots} (= 11.7\dots)$ oe			M1 complete method to find AC showing correct order of operations
	eg [[AD =] $\frac{11.7 \sin 35}{\sin 82}$ (= 6.77..) or [DC =] $\frac{11.7 \times \sin 63}{\sin 82}$ (= 10.5..)oe (where "82" = $180 - 98$, "63" = $180 - "82" - 35$)			M1 correct calculation for AD or DC dep on 1 st M1 and 2 nd M1
	eg [AD =] $\frac{11.7 \sin 35}{\sin 82}$ and [DC =] $\frac{11.7 \sin 63}{\sin 82}$ oe or [AD =] $\frac{11.7 \sin 35}{\sin 82}$ and [DC =] $\sqrt{11.7^2 + 6.77^2 - 2 \times 11.7 \times 6.77 \times \cos 63}$ [DC =] $\frac{11.7 \sin 63}{\sin 82}$ and [AD =] $\sqrt{11.7^2 + 10.5^2 - 2 \times 11.7 \times 10.5 \times \cos 35}$ Where "63" = $180 - "82" - 35$			M1 correct calculations for AD and DC (AD = 6.77... DC = 10.5...) dep on 1 st M1 and 2 nd M1
	Working not required, so correct answer scores full marks (unless from obvious incorrect working)	32.8		A1 accept 32.7 – 32.9
				Total 6 marks



Q7.

Q	Working	Answer	Mark	Notes	
	Angle $DBA = 43^\circ$ or Angle $DAB = 90^\circ$			M1	
	$180 - 90 - 43 (=47)$			M1	
				A1	
		47 with reasons	5	B2	for full reasons Angles in the same segment are equal; Angle in a semi-circle is a right angle Angles in a triangle add up to 180° (B1 for a correct and relevant reason using a circle theorem)
					Total 5 marks

Q8.

Q	Working	Answer	Mark	Notes	
	$3 \times (3 + 8.5) = 5 \times PR$ or $3 \times (3 + 8.5) = 5 \times (5 + PQ)$			M1	
	$(3 \times (3 + 8.5)) \div 5 - 5$			M1	for a complete method for PQ
		1.9	3	A1	
					Total 3 marks



Q9.

Q	Working	Answer	Mark	Notes
	$(PT =) \frac{12 \times 4}{3} (= 16)$		3	M1 NB: 16 from $12 + 4$ is incorrect working
	$(r =) ("16" + 3) \div 2$			M1
		9.5		A1 oe
				Total 3 marks

Q10.

Q	Working	Answer	Mark	Notes
	$(\angle AOC =) 132 \times 2 (= 264)$		3	M1 for method to find angle at the centre. Do not award this mark if contradicted on the diagram eg if obtuse AOC is labelled as 264
	eg $\frac{"264"}{360} \times 2 \times \pi \times 8.5 (= 39.1... \text{ or } \frac{187}{15} \pi)$ or $2 \times \pi \times 8.5 - \frac{360 - "264"}{360} \times 2 \times \pi \times 8.5 (= 39.1... \text{ or } \frac{187}{15} \pi)$ or $\frac{"264"}{360} \times 2 \times \pi \times 8.5 + 2 \times 8.5$ or $2 \times \pi \times 8.5 - \frac{360 - "264"}{360} \times 2 \times \pi \times 8.5 + 2 \times 8.5$			M1 for a method to find the length of arc AC or perimeter of the sector – allow use of their AOC as long as clearly labelled
	<i>Correct answer scores full marks (unless from obvious incorrect working)</i>	56.2		A1 accept 56.1 – 56.2
				Total 3 marks